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DETERMINATION OF THE THERMAL CONDUCTIVITY IN SOLIDIFYING INGOTS

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The problem of identification of the effective thermal conductivity in a solidifying ingot is considered on the basis of experimental data. The possibility of its solution with identifiability conditions obtained and implemented in the present work is determined.

The quality of mathematical modeling of heat- and mass-transfer processes greatly depends on solution of the problem of parametric identification and is mainly determined by the exactness of coefficients which enter into the equations of convection and heat and mass transfer. The reliable values of these coefficients can be obtained only by solution of corresponding inverse problems [1, 2]. We consider the problem of identifiability of the effective thermophysical parameters which allow one to abandon calculation of convection equations and thus to substantially simplify computation schemes. The suggested algorithm of identification is constructed on the basis of the classical theories of solution of direct problems and optimization methods.

As an example, we consider the one-dimensional process of heat exchange between a melt and a casting-mold wall and the surrounding medium at the initial stage of solidification:

$$C\rho \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) = 0 \,, \quad (t, x) \in (t_a, t_b] \times (x_0, x_1) \,. \tag{1}$$

Since the effective thermal conductivity λ_e in the melt at $x \in (x_0, x_1)$ is assumed to be uniform and in the casting-mold wall at $x \in (x_1, x_2)$ to be known and equal to λ_1, we can write $\lambda(t, x) = \lambda_1 + [\lambda_e(t) - \lambda_1]\,\Theta(x - x_1)$. In this case, Eq. (1) is transformed to the form [3]

$$C\rho \frac{\partial T}{\partial t} - \lambda \frac{\partial^2 T}{\partial x^2} - (\lambda_e - \lambda_1)\,\delta\,(x_1 - x)\frac{\partial T}{\partial x} = 0 \,. \tag{2}$$

The boundary and initial conditions are as follows:

$$\left.\frac{\partial T}{\partial x}\right|_{x_0} = 0 \,, \quad \lambda_1 \left.\frac{\partial T}{\partial x}\right|_{x_2} = -\,\alpha\,(T - T_{out}) \,, \quad T\big|_{t_a} = T_a \,.$$

We will evaluate the quality of identification of $\lambda_e(t)$ by the difference between the experimentally determined temperature of the melt $T_g(t)$ at a certain point $x_g \in (x_0, x_1)$ and the temperature $T(t, x_g)$ calculated by model (2) at the same point:

$$J\,(\lambda_e) = \int_{t_a}^{t_b} [T\,(t, x_g) - T_g\,(t)]^2\, dt \,. \tag{3}$$

Using a direct extremum approach [4], we construct the iteration algorithm of search for the optimum value of the parameter $\lambda_e(t)$:

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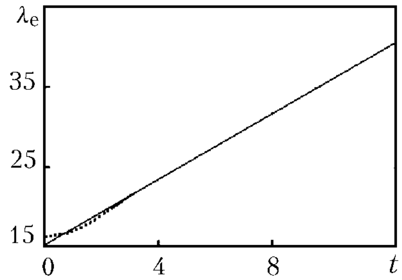

Fig. 1. Testing of the identification algorithm: solid line) exact value of $\lambda_e(t)$; dashed line) $\lambda_e(t)$ identified by algorithm (4) with condition (7). λ_e, W/(m·K); t, sec.

$$\lambda_e^{k+1}(t) = \lambda_e^k(t) - b^k \beta(t) \nabla J(\lambda_e^k; t), \quad b^k > 0, \quad k = 0, 1, 2, \dots . \tag{4}$$

Here the gradient of functional (3) is

$$\nabla J = \frac{\partial f}{\partial x}\frac{\partial T}{\partial x}, \quad (t, x) \in (t_a, t_b) \times x_* . \tag{5}$$

It is calculated at a certain point of the melt x_* in terms of the solution $f(t, x)$ of the conjugate problem

$$C\rho \frac{\partial f}{\partial t} + \lambda \frac{\partial^2 f}{\partial x^2} + (\lambda_e - \lambda_1)\, \delta(x - x_1) \frac{\partial f}{\partial x} - 2(T - T_g)\, \delta(x - x_g) = 0 . \tag{6}$$

The boundary and initial conditions for (6) have the form

$$\left.\frac{\partial f}{\partial x}\right|_{x_0} = 0 , \quad \left.\frac{\partial f}{\partial x}\right|_{x_2} = -\alpha f , \quad f\big|_{t_b} = 0 .$$

A distinctive feature of the direct extremum approach to solution of the identification problem (2), (3) with algorithm (4), for example, as compared to [1], is that it does not require either preliminary digitization of the problem or expansion of the sought function $\lambda_e(t)$ in series.

We now pass to an analysis of the identifiability of the considered problem in the context of [5]. The direct problems (2) and (6) are of parabolic form. It is well known that using corresponding difference schemes one can obtain a correct solution of such problems [6]. It is seen from expression (6) that a nontrivial (nonidentically zeroth) solution of the conjugate problem is possible only when $x_g < x_* < x_1$. This follows from the presence of the δ function in the free term of the equation. It is precisely this term that is the source of such a solution. In integration of (6), the δ function is transformed to the Θ function, which differs from zero to the right of x_g. Consequently, (6) will have a non-zero solution f only to the right of x_g.

Thus, the effective value of the thermal conductivity λ_e in model (2) is identifiable by the objective functional (3) when

$$x_g < x_* < x_1 . \tag{7}$$

Figure 1 gives results of testing of the direct algorithm (4) with condition (7). Test calculations were organized according to the following scheme. A certain value of the effective thermal conductivity was assigned (solid line in Fig. 1) and the initial problem (2) was solved. The determined temperature was taken as an experimental T_g based on which we solved by algorithm (4) the inverse problem of determination of $\lambda_e(t)$, minimizing functional (3).

During only 21 iterations, at $\beta(t) = 0.2\lambda_e^0(t)/\left|\nabla J(\lambda_e^0; t)\right|$ and $\lambda_e^0(t) = 5$ W/(m·K) we obtained the solution (dashed curve in Fig. 1), which virtually coincided with the exact value of the sought parameter. We note that calcu-

Fig. 2. Results of identification of the effective thermal conductivity (a) and the temperature of the ingot at the point x_g calculated according to model (2) (b). Points) experimental temperature. λ_e, W/(m·K); T, K; t, sec.

lations without account for the identifiability conditions (7) led to incorrect results, and neglect of the parameter β, which controls the direction of descent to min J with account for the sensitivity of J to λ_e, drastically decreased the rate of convergence and did not allow the exact solution during a finite number of iterations. The obtained test calculations demonstrate the reliability and high efficiency of the method.

Figure 2 gives results of identification of the effective thermal conductivity λ_e by algorithm (4) during 25 iterations for the actual process of solidification of a steel melt in a cylindrical casting mold on the basis of experimental data [7]. The thickness of the wall of the casting equipment is $x_2 - x_1 = 0.03$ m, the ingot radius is $x_1 - x_0 = 0.08$ m, and the coordinate of the temperature-sensitive element is $x_g = 0.04$ m. The adopted ratio of the linear dimensions of the ingot and use of the effective value λ_e allow one to use the one-dimensional model of the process of solidification [7]. The parameters are as follows: $\alpha = 580$ W/(m^2·K), $C = 650$ J/(kg·K), $\rho = 6950$ kg/m^3, $T_{out} = 300$ K, and $T_a = 1673$ K.

The value of the function $\lambda_e(t)$ found by algorithm (4) for the mathematical model (2) with the identifiability condition (7) is shown in Fig. 2a. The results are obtained at the initial approximation $\lambda_e^0(t) = 5$ W/(m·K) to which there corresponded the maximum temperature difference $\max\limits_t \left| T(t, x_g) - T_g(t) \right| = 224$ K. For the $\lambda_e(t)$ found it was only 4 K. The results of the experiment and of the calculation of the temperature according to the model which are given in Fig. 2b indicate a high accuracy of the modeling of solidification processes.

Use of direct extremum algorithms of identification with the suggested analysis of identifiability allows one to efficiently solve the inverse problems of identification employing only traditional theories of solution of direct problems.

NOTATION

C and ρ, heat capacity and density of the melt; λ, thermal conductivity; α, heat-transfer coefficient; T, temperature; x, coordinate; x_0, center of the ingot; x_1, "ingot–equipment" boundary; x_2, "equipment–surrounding medium" boundary; x_g, coordinate of the temperature-sensitive element; t, time; Θ and δ, Heaviside function and delta function; J, objective functional; ∇J, gradient of the objective functional (linear functional); b^k, step of the method; β, parameter of control of the convergence of the optimization algorithm; f, conjugate state (linear functional). Subscripts: e, effective; out, outer medium; g, gauge; a and b, initial and finite time of the process; *, identifiability region. Superscripts; 0, initial approximation; k, iteration number.

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